Chiral logs with staggered fermions*

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We compute chiral logarithms in the presence of "taste" symmetry breaking of staggered fermions. The lagrangian of Lee and Sharpe is generalized and then used to calculate the logs in π and K masses. We correct an error in Ref. [1]; the issue turns out to have implications for the comparison with simulations, even at tree level. MILC data with three light dynamical flavors can be well fit by our formulas. However, two new chiral parameters, which describe $\mathcal{O}(a^2)$ hairpin diagrams for taste-nonsinglet mesons, enter in the fits. To obtain precise results for the physical $\mathcal{O}(p^4)$ coefficients, these new parameters will need to be bounded, at least roughly.

It has become clear that simulation at rather light quark mass is crucial for accurate determination of physical parameters, *e.g.*, heavy-light decay constants [2]. Since staggered fermions provide the fastest known method of simulating low-mass quarks, the systematic effects associated with this fermion choice should be studied.

We adopt the following nomenclature: A single staggered field describes 4 equivalent "tastes" of quarks in the continuum limit; taste symmetry is broken at $\mathcal{O}(a^2)$. We use the word "flavor" for different staggered fields; lattice flavor symmetry is exact for equal masses. MILC's improved staggered ("a^2-tad") simulations [3] use three fields ("u, d, s") and reduce the tastes to one per flavor by taking $\sqrt[4]{\mathrm{Det}}$. We call these "$2+1$" flavor simulations since we take $m_u = m_d \equiv m_\ell$.

MILC data for m_π^2 *vs.* quark mass show clear deviations from linearity, as expected from chiral logarithms. However, the detailed behavior at $a \approx 0.13$ fm does not agree with continuum chiral perturbation theory (χPT) [4]. Figure 1 shows a fit to the continuum forms for $m_X^2/(m_1+m_2)$, where $X = \pi$ or K, and m_1, m_2 are quark masses. (We define "pions" to have $m_1 = m_2$ and "kaons" to have $m_2 \approx m_s^{\rm phys}$ and $m_1 \neq m_2$.) The fit is very poor, with a confidence level of 5×10^{-5}.

To get good fits, one needs to include the $\mathcal{O}(a^2)$ taste-breaking effects into the chiral calculations, *i.e.*, we need "staggered chiral perturbation theory" (SχPT). Such calculations start with the chiral lagrangian of Lee and Sharpe [5] for 1 staggered flavor (4 tastes). One defines

$$\Sigma(x) \equiv \exp(i\phi/f)\ ; \quad \phi = \phi_5\xi_5 + \phi_\mu\xi_\mu + \cdots, \tag{1}$$

where ϕ is a 4×4 matrix of pseudoscalar mesons of various tastes, and $\xi_5, \xi_\mu, \ldots$ are taste matrices. The Lee-Sharpe lagrangian is then

$$\mathcal{L} = \frac{f^2}{8}\mathrm{tr}(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) - \frac{\mu m f^2}{4}\mathrm{tr}(\Sigma+\Sigma^\dagger) + a^2\mathcal{V} \tag{2}$$

The taste-breaking potential $\mathcal{V}$ has various operators with explicit taste matrices: $-\mathcal{V} =$

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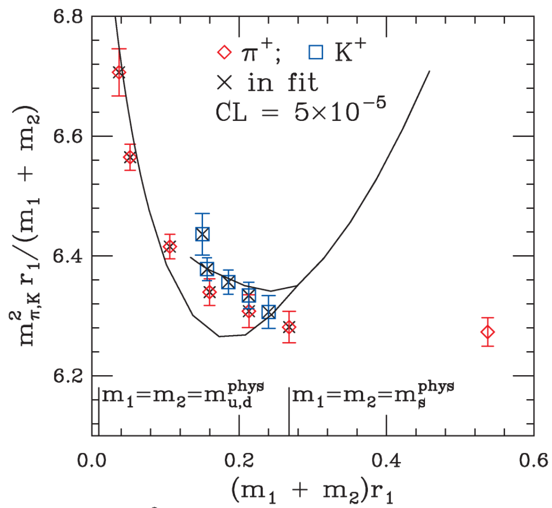

Figure 1. $m^2_{\pi,K}/(m_1+m_2)$ *vs.* m_1+m_2, in units of the potential scale r_1, for $2+1$ flavor lattices at $a \approx 0.13$ fm. The fit is to the continuum chiral log forms. The upper branch for $(m_1+m_2)r_1 \sim 0.2$ uses the kaon form; the rest of the curve is the pion form.

$\sum_{i=1}^{6} C_i O_i$. For example,

$$O_2 = \tfrac{1}{2}[\mathrm{tr}(\Sigma\Sigma) - \mathrm{tr}(\xi_5\Sigma\xi_5\Sigma) + \mathrm{h.c.}] \quad (3)$$
$$O_5 = \tfrac{1}{2}[\mathrm{tr}(\xi_\nu\Sigma\xi_\nu\Sigma^\dagger) - \mathrm{tr}(\xi_\nu\xi_5\Sigma\xi_5\xi_\nu\Sigma^\dagger)]\ . \quad (4)$$

To get NLO chiral results relevant to $2+1$ simulations, we follow a three step procedure:

1. Generalize the Lee-Sharpe lagrangian to the 3-flavor case, *i.e.*, include three lattice staggered fields (u,d,s), each with four tastes, and masses $m_u = m_d \equiv m_\ell \neq m_s$. This is an "$8+4$" theory.

2. Compute the desired quantities (here, m^2_π and m^2_K) at one loop in the $8+4$ case.

3. Adjust the $8+4$ answer "by hand" to correspond to the $2+1$ case of interest by identifying the chiral contributions that correspond to n virtual quark loops and dividing them by 4^n.

Step 1 turns out to be non-trivial: Ref. [1] employed Fierz transformations to simplify the flavor structure of the generalized potential $\mathcal{V}$. However, Ref. [5] already used the Fierz freedom in deriving $\mathcal{V}$. The net result is that the generalized operators O_2 and O_5 are incorrect in Ref. [1].

There are actually two operators that correspond to each of O_2 and O_5:

$$O_{21} = \tfrac{1}{4}[\mathrm{Tr}(\xi_\mu\Sigma)\mathrm{Tr}(\xi_\mu\Sigma) + \mathrm{h.c.}]$$
$$O_{22} = \tfrac{1}{4}[\mathrm{Tr}(\xi_\mu\xi_5\Sigma)\mathrm{Tr}(\xi_5\xi_\mu\Sigma) + \mathrm{h.c.}]$$
$$O_{51} = \tfrac{1}{2}\mathrm{Tr}(\xi_\mu\Sigma)\mathrm{Tr}(\xi_\mu\Sigma^\dagger)$$
$$O_{52} = \tfrac{1}{2}\mathrm{Tr}(\xi_\mu\xi_5\Sigma)\mathrm{Tr}(\xi_5\xi_\mu\Sigma^\dagger)\ . \quad (5)$$

In the absence of flavor indices, the combinations $O_{21}+O_{22}$ and $-O_{51}+O_{52}$ can be Fierzed into Lee-Sharpe O_2 and O_5, respectively; the other linear combinations turn into other O_i.

The new operators have a surprising effect: they generate hairpin (disconnected) diagrams for taste-nonsinglet (but flavor neutral) mesons. For taste vector and taste axial vector mesons, this occurs at chiral tree level. The magnitudes of the hairpins are

$$\delta'_V \equiv \frac{16a^2}{f^2}(C_{21}-C_{51});\ \delta'_A \equiv \frac{16a^2}{f^2}(C_{22}-C_{52}) \quad (6)$$

for vector and axial taste, respectively.

The disconnected hairpin diagrams have not been included in any simulations to date of taste-nonsinglet mesons. Thus the Lee-Sharpe lagrangian does not apply to such simulations: The chiral lagrangian requires more than one flavor to describe particles, *e.g.*, $\pi^+ = u\bar{d}$, that have no disconnected contributions.

At 1-loop, we need to iterate three kinds of hairpins on flavor-neutral internal lines: the standard anomaly hairpin (m^2_0) for taste singlets, and the new hairpins for taste vector and axial vector. Unlike m^2_0, $\delta'_{V,A}$ cannot be taken to infinity. We therefore have to rediagonalize the mass matrix in the taste V, A channels. We call, for example, the mass eigenstates in the flavor-neutral, taste vector channel π^0_V, η_V and η'_V. Techniques in Ref. [6] are very useful for reexpressing the iterated propagator as a sum of simple poles.

Adjustment to go from the $8+4$ to the $2+1$ theory is easy. Every hairpin interaction on a meson line after the first introduces an additional virtual quark loop. Therefore, if $F(\Delta)$ is the sum of hairpin diagrams, just take $F(\Delta) \to 4F(\frac{\Delta}{4})$ for $\Delta = \delta'_V$, δ'_A or δ (where $\delta \equiv m^2_0/(24\pi^2 f^2)$).

Additional diagrams that were identified as valence diagrams in Ref. [1] turn out to be *disconnected* and contribute only to flavor-neutral correlators.

We define $\beta_{K_5^+}$ to be the chiral log term for the Goldstone K^+ mass $(m^{1-\mathrm{loop}}_{K_5^+})^2/(\mu(m_\ell+m_s)) =$

$1 + \beta_{K_5^+}/(16\pi^2 f^2) + \cdots$, and similarly for $\beta_{\pi_5^+}$. Our results are then:

$$\beta_{\pi_5^+} = \frac{2\delta'_V}{m^2_{\eta'_V} - m^2_{\eta_V}} \left[\frac{m^2_{\eta'_V} - m^2_{S_V}}{m^2_{\eta'_V} - m^2_{\pi_V}} m^2_{\eta'_V} \ln m^2_{\eta'_V} - \frac{m^2_{\eta_V} - m^2_{S_V}}{m^2_{\eta_V} - m^2_{\pi_V}} m^2_{\eta_V} \ln m^2_{\eta_V} \right] - 4m^2_{\pi_V} \ln m^2_{\pi_V}$$
$$+ (V \to A) + m^2_{\pi_I} \ln m^2_{\pi_I} - \tfrac{1}{3} m^2_{\eta_I} \ln m^2_{\eta_I}$$
$$\beta_{K_5^+} = 2\delta'_V \left[\frac{m^2_{\eta'_V} \ln m^2_{\eta'_V} - m^2_{\eta_V} \ln m^2_{\eta_V}}{m^2_{\eta'_V} - m^2_{\eta_V}} \right] + (V \to A) + \tfrac{2}{3} m^2_{\eta_I} \ln m^2_{\eta_I} \ . \qquad (7)$$

Here S is the $s\bar{s}$ meson. The continuum result is in the taste-singlet channel (I); the remainder vanishes in the limit $\delta'_{V,A} \to 0$.

With $\delta'_{V,A}$ as free parameters, these results give excellent fits to the MILC data. However, the parameters are poorly constrained: δ'_V and δ'_A wander off in opposite directions to large values (an order of magnitude larger than known taste-violating terms in the π^+ sector). On the other hand, if we fix δ'_V or δ'_A to be of reasonable magnitude, the fits find reasonable values for the other parameter, too, and the confidence levels are still excellent. Figure 2 shows the fit with $\delta'_A r_1^2$ fixed to -0.1; $\delta'_V r_1^2$ is then found to be 0.25(11). Based on the measured taste violations, the natural size for these parameters is $\delta' r_1^2 \sim 0.2$.

We conclude that SχPT is necessary to fit current staggered lattice data. The taste-violating hairpins introduce two new free parameters ($\delta'_{V,A}$) into the chiral theory; other taste violations, which split flavor-nonsinglet mesons, are not free parameters in the fits because they are measured directly in simulations. Unfortunately, the hairpin parameters are not well constrained, at least at present. It does however appear to us that even a rough constraint on $\delta'_{V,A}$ (*e.g.*, demanding that they be no more than 3 times the known taste-violating terms) will be sufficient to determine p^4 analytic coefficients (Gasser-Leutwyler "L_i") with good precision.

We may be able to constrain the hairpin parameters in a variety of ways. Additional SχPT calculations are underway [7] that generalize these results to the quenched and partially quenched cases, and extend them to pseudoscalar decay constants (of both light-light and heavy-light mesons). With these calculations in hand, the direct constraints from all the fits may prove sufficient. Another approach is to compute the four-quark, taste-violating operators perturbatively [8]; the hairpin parameters may then be estimated by vacuum saturation or lattice calculation of the matrix elements. Finally, a direct lattice evaluation of the disconnected hairpin graphs may be possible.

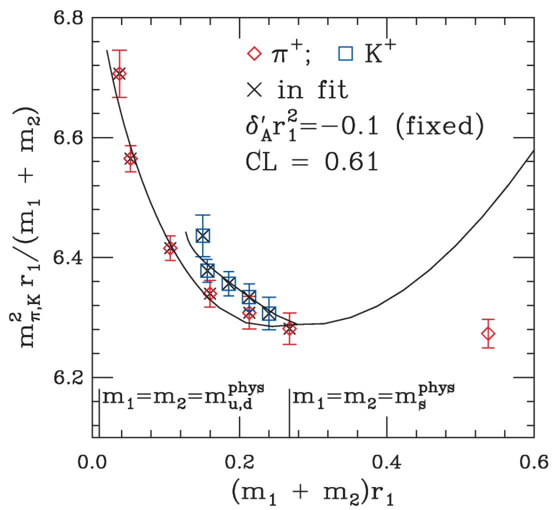

Figure 2. Same as Fig. 1, but using the SχPT forms, eq. (7). The parameter δ'_A is held fixed to a reasonably small value in the fit.

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